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The Resonant Frequency of Rectangular Interdigital Filter Elements

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***Abstract*—A procedure is given for the computation of the resonant frequency of loosely coupled interdigital resonators with rectangular cross section. The procedure is based on the use of Getsinger's fringing capacitance data [1]. The accuracy of the method was verified experimentally and found to be approximately 1 percent for a 2-percent bandwidth interdigital linear-phase filter.**

I. Introduction

Certain microwave structures, such as interdigital filters, are constructed using an array of parallel coupled rectangular cross-sectional resonators [1]. The side view of an interdigital resonator is shown in Fig. 1 and a plan view in Fig. 2. The geometry of the resonator end is shown in Fig. 3. The resonator has width w, thickness t, and length l'. It is symmetrically enclosed in a cavity of length l, formed by two parallel plates with ground plane spacing b. The cavity is filled with a homogeneous dielectric of relative permittivity ϵ_r. One end of the resonator is short circuited by the vertical wall of the cavity, while the open end is separated from the other vertical wall by a gap of length g.

It is assumed that only the TEM mode propagates, and that the interdigital resonator can be represented by the equivalent circuit of Fig. 4 where Z_0 is the characteristic impedance of the rectangular cross-sectional resonator at the center frequency, and C_g is a lumped capacitance due to the gap. Z_0 is determined by the cross-sectional dimensions, w, t, and b, and the spacing of adjacent resonators. In practice, once w, t, and b have been selected, the problem in resonator design is to find the gap length g, which yields the correct gap capacitance C_g, for a specified resonant frequency f_0.

The problem of computing the gap capacitance has been addressed by Nicholson [2] and Khandelwal [3]. Nicholson's procedure is for circular cross-sectional resonators. Khandelwal's more elaborate procedure is useful for general cross sections. Incidentally, Khandelwal's procedure for computing the fringing capacitance between the resonator tip and the end, top, and bottom plates [3, fig. 2] is incorrect because of the addition of $2C'_{fe}$ to $2C'_{fo}$. Getsinger's odd-mode capacitance $2C'_{fo}$ is the total fringing capacitance to ground, and includes the effect of top and bottom plates as well as the end plate [1, fig. 6(a)].

The procedure described here is applicable to loosely coupled resonators of rectangular cross section, is simple to use, and has given good results in the design of a narrow-band interdigital linear-phase filter.

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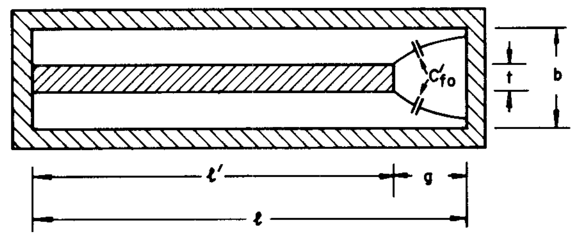

Fig. 1. Side view of an interdigital resonator.

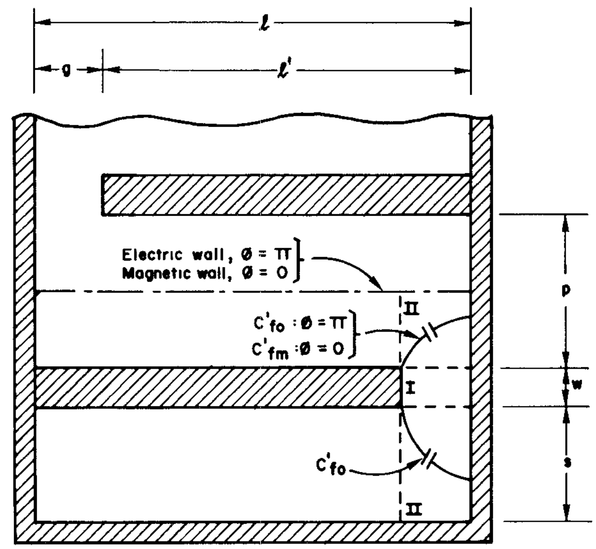

Fig. 2. Plan view of an end resonator showing boundary conditions.

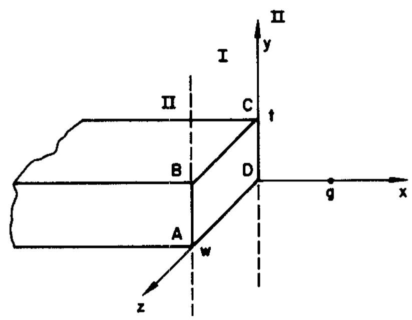

Fig. 3. Geometry of the resonator end.

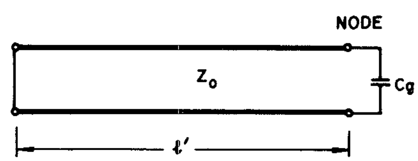

Fig. 4. Resonator equivalent circuit.

II. The Gap Capacitance

The cavity length is

$$l = \lambda_0 / (4\sqrt{\epsilon_r}) \tag{1}$$

where λ_0 is the free-space wavelength at the desired resonant frequency f_0.

At resonance, the admittance at the node in Fig. 4 is zero. Inspection of the equivalent circuit shows that the resonance condition is satisfied when

$$2\pi f C_g|_{f=f_0} = 1/\{Z_0 \tan(2\pi f(l-g)/v)\}|_{f=f_0} \quad (2)$$

where $v = c/\sqrt{\epsilon_r}$ is the relative velocity of propagation in the medium.

Equation (2) can be solved graphically or numerically for the gap length g if the gap capacitance C_g can be expressed as a function of g. The gap capacitance is a function of the geometry shown in Fig. 3. As mentioned, the ground plane spacing b, the bar width w, and the thickness t, are determined by the desired characteristic impedance Z_0. When these parameters have been selected, the gap width g is the variable which determines the gap capacitance C_g, and thus the resonant frequency f_0. The gap capacitance can be expressed as

$$C_g = C_I + C_{II}. \quad (3)$$

C_I is defined as the capacitance in the gap region I formed by the planes $z=0$, $z=w$, $x=0$, the vertical wall ($x=g$), the top plate, and the bottom plate. C_{II} is the capacitance in the remaining gap region. C_I is given exactly by Getsinger's odd-mode fringing capacitance per unit length C'_{fo}, as

$$C_I = 2wC'_{fo}(2g/b, t/b). \quad (4)$$

In contrast to C_I, which is composed of the capacitances due to the two horizontal edges BC and AD plus the parallel plate capacitance due to the resonator's face, C_{II} is composed of the capacitances per unit length due to the vertical edges AB and CD, namely C'_{fv_1}, and C'_{fv_2}, and C_{fc}, the capacitance due to each of the four corners. Thus

$$C_{II} = t\left(C'_{fv_1} + C'_{fv_2}\right) + 4C_{fc}. \quad (5)$$

In principle, the vertical edge capacitances C'_{fv_1}, and C'_{fv_2} can be found as follows for an interdigital filter. For an end resonator, as depicted in Fig. 2, one of the vertical edges will be adjacent to a vertical electric wall due to the cavity side wall, and the appropriate vertical edge fringing capacitance per unit length is given exactly in terms of Getsinger's odd-mode capacitance as

$$C'_{fv} = C'_{fo}(2g/(2s+w), w/(2s+w)) - \epsilon\frac{w/2}{g} \quad (6)$$

where s is the distance from the resonator edge to the end wall, and w the resonator width. The second term is the parallel plate capacitance due to the half of the resonator face associated with the vertical edge, and must be subtracted from C'_{fo} since the face capacitance is included in (4) for C_I.

For the other vertical edge, the vertical boundary condition is defined by the physical spacing and electric excitation of the adjacent resonator. In the equivalent circuit of an interdigital filter, all resonators and unit elements are one quarter-wavelength long at the center frequency and consequently the voltages on adjacent resonators have phase difference $\phi = \pi/2$ at the center frequency. Therefore, the vertical boundary is neither an electric nor a magnetic wall and the resulting fringing capacitance must be found by decomposing the excitation of the two resonators into the even mode $\phi = 0$, which results in a magnetic wall halfway between the resonators, and the odd mode $\phi = \pi$, which results in an electric wall halfway between the resonators, then finding the corresponding fringing capacitances C'_{fm} and C'_{fo}, and finally combining them appropriately. Using this approach, the vertical edge fringing capacitance per unit length at the center frequency $\phi = \pi/2$ can be expressed as

$$C'_{fv} = \frac{1}{2}\left\{C'_{fm}(2g/(p+w), w/(p+w)) + C'_{fo}(2g/(p+w), w/(p+w))\right\} - \epsilon\frac{w/2}{g}. \quad (7)$$

In this expression C'_{fo} is Getsinger's odd-mode fringing capacitance per unit length from the vertical edge and associated half of the resonator face and arises when $\phi = \pi$. Note that p is the edge-to-edge spacing between adjacent resonators. C'_{fm} is the fringing capacitance for even-mode excitation $\phi = 0$ of the adjacent resonators and no straightforward method for obtaining this capacitance appears to have been published, although Cohn's data [4] for semi-infinite coplanar plates could possibly be adapted.

The purpose of the paper is to present a simple method, yet sufficiently accurate for practical filter design, of estimating the gap spacing g for a particular center frequency. Accordingly, the following approximations are proposed to simplify computation of C_{II}. Firstly, that the corner capacitances in (5) are negligible, thus

$$C_{fc} \cong 0. \quad (8)$$

Secondly, that C'_{fm} is approximately equal to C'_{fo} in (7), in which case

$$C'_{fv} \cong C'_{fo}(2g/(p+w), w/(p+w)) - \epsilon\frac{w/2}{g} \quad (9)$$

for a vertical edge adjacent to another resonator. The second approximation can be justified for loosely coupled resonators by the intuitive argument that the end wall will play the dominant role in the gap field distribution, whereas the distant electric or magnetic walls will play a relatively insignificant role. Under these assumptions, it follows from (3)–(9) that the gap capacitance for an end resonator is approximated by

$$C_g \cong 2wC'_{fo}(2g/b, t/b) + tC'_{fo}(2g/(2s+w), w/(2s+w)) + tC'_{fo}(2g/(p+w), w/(p+w)) - \epsilon\frac{wt}{g} \quad (10)$$

with the symbols defined in Figs. 1 and 2.

Similarly, for a resonator located between two adjacent resonators, the gap capacitance is approximated by

$$C_g \cong 2wC'_{fo}(2g/b, t/b) + tC'_{fo}(2g/(p_1+w), w/(p_1+w)) + tC'_{fo}(2g/(p_2+w), w/(p_2+w)) - \epsilon\frac{wt}{g} \quad (11)$$

where p_1 and p_2 are the inter-conductor spacings associated with each of the vertical edges.

TABLE I
THE THEORETICAL AND EXPERIMENTAL RESONANT FREQUENCIES OF AN INTERDIGITAL RESONATOR

g (mm)	C_I/ε (mm)	C_{II}/ε (mm)	C_g/ε (mm)	f_o (MHz) Theory	f_o (MHz) Measured	Difference %
3.0	28.2	8.8	37.0	1445	1430	1%
4.5	21.1	7.5	28.6	1522	1510	0.8%

$b=15$mm $t=6$mm $t/b=0.4$mm $\ell=50$mm
$w=8.8$mm $p_1=p_2=p=17$mm $\varepsilon_r=1$ $Z_o=50\Omega$

Because a simple, closed-form expression for C'_{fo} as a function of gap length g is not available, the value of g which satisfies (2) has to be found iteratively as follows. For a particular value of g, the odd-mode fringing capacitances C'_{fo}, which occur in (10) or (11), are found from Getsinger's Fig. 4, and the corresponding gap capacitance C_g, calculated. Next, the frequency f_0, which satisfies (2), is found graphically or numerically. The process is repeated to produce a graph of resonant frequency f_0 as a function of gap length g. The value of g which corresponds to the desired resonant frequency can then be found from the graph.

III. Experimental Results

The validity of the assumptions and approximations in the procedure have been tested for a resonator located between two adjacent resonators, in a 2-percent bandwidth interdigital linear-phase filter of degree 6. The measured resonant frequency was compared to the theoretical frequency, computed using (11), for two gap lengths and Table I shows that good agreement was obtained between theory and experiment.

IV. Conclusion

A simple procedure has been presented for computing the gap width of the loosely coupled rectangular cross-sectional interdigital resonator to obtain a specified resonant frequency. The method uses a simple formula for the gap capacitance of the resonator, based on Getsinger's odd-mode fringing capacitance data [1].

Experimental results suggest that the accuracy of the method is of the order 1 percent. From Table I it is observed that the theoretical resonant frequency is higher than the measured frequency, which indicates that the theoretical gap capacitance C_g is less than the actual capacitance. This could imply that the corner capacitances are not negligible, as assumed in approximation (8).

The technique is expected to give best results for structures that use loosely coupled bars, such as narrow-band filters. Tightly coupled bars would have different even- and odd-mode fringing capacitances, thus invalidating approximation (9). However, the procedure should be adequate for most practical filter designs, since tightly coupled bars are usually associated with wide-band filters which are relatively insensitive to deviations from the ideal circuit element values. A practical resonator design approach is to design the gap width so that the theoretical resonant frequency is 1–2 percent higher than the desired frequency, and to provide a capacitive tuning screw at the gap. The gap width should be slightly larger than required when the tuning screw is flush with the cavity wall. Therefore, the resonant frequency can be reduced to the desired value by introducing the tuning screw into the gap, thus reducing the effective gap width and increasing the gap capacitance.

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A Design Method for Noncommensurate Broad-Band Matching Networks

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Abstract —**A simple design method for broad-band ladder network impedance transformers having noncommensurate section lengths and predictable passband response is presented. Limitations on section impedances imposed by constructional constraints are more easily met than with commensurate networks, and harmonically related passbands are largely avoided. An example is presented.**

I. Introduction

The short-step impedance transformer [1] and other broad-band transmission-line networks based on rational insertion loss functions employ commensurate (equal) element length sections. As a consequence of this constraint, the frequency response is highly periodic. Moreover, having specified the desired frequency response, overall network length and circuit topology (i.e., number of series/shunt elements etc.), the designer has little flexibility with the range of element characteristic impedances required.

For example, in a particular application a designer might consider the use of a series-cascaded commensurate network because of its relative design and constructional simplicity. Having specified the passband response and overall network length, the minimum number of sections needed and thereafter the section impedances can be determined. Because of the short element lengths, a wide range of section impedances usually results. The designer must then determine whether such a design can be realized in practice, in the transmission-line type desired. (A typical range feasible in microstrip, for example, is 20–110 Ω, while in slotline 55–300 Ω, or coax 10–100 Ω.) If the design is not practical, then using commensurate elements, only a network of greater complexity and/or length will provide the desired passband performance.

On the other hand, noncommensurate networks have the advantage of allowing greater design flexibility because the constraint that all elements have the same electrical length is removed. A further advantage of such designs is that the periodic recurrence of higher order passbands is largely avoided. However, the circuit transfer function can no longer be described in terms of rational functions. To date, no general theory has been presented for the synthesis of noncommensurate circuits with prescribed gain functions.

In this paper, a design procedure is outlined enabling a noncommensurate network to be derived from a commensurate prototype. The procedure involves the use of a transformation which keeps the fundamental passband frequency response of the derived noncommensurate circuit almost identical to that of the commensurate prototype. This is achieved while providing the designer with greater flexibility in the choice of element impedance levels.

II. Design Procedure

Having considered the conventional commensurate design and found it to be unsuitable, a noncommensurate network may be derived by considering pairs of sections of the commensurate

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